

Exact Solitary-wave Solutions for the General Fifth-order Shallow Water-wave Models

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We perform a computerized symbolic computation to find some general solitonic solutions for the general fifth-order shallow water-wave models. Applying the tanh-typed method, we have found certain new exact solitary wave solutions. The previously published solutions turn out to be special cases with restricted model parameters.

We investigate the generalized fifth-order shallow water-wave model which describes certain physically-interesting (1+1)-dimensional waves but is not integrable by the conventional methods [1]:

$$v_t + \alpha v_{xxxxx} + \mu v_{xxx} + \gamma \partial_x [2vv_{xx} + v_x^2] + 2qv v_x + 3rv^2 v_x = 0, \quad (1)$$

where α , μ , γ , q , and r are model parameters, v is a real scalar function of the two independent variables, x and t , and the subscripts denote partial derivatives.

We apply the tanh-typed method [2–4] with symbolic computation to (1) and assume that the physical field $v(x, t)$ has the form

$$v(x, t) = \sum_{n=0}^N A_n(t) \cdot \tanh^n [\mathcal{G}(t)x + \mathcal{H}(t)], \quad (2)$$

where N is an integer determined via the balance of the highest-order contributions from the linear and non-linear terms of (1) as $N=2$, while $A_N(t)$, $\mathcal{G}(t)$, and $\mathcal{H}(t)$ are the non-trivial differentiable functions to be determined.

With the symbolic computation package *Maple* we introduce Ansatz (2) together with the above conditions into (1) and collect the coefficients of like powers of $\tanh$:

$$(\tanh^7): -6A_2(t)\mathcal{G}(t) \cdot [rA_2(t)^2 + 16\gamma\mathcal{G}(t)^2A_2(t) + 120\alpha\mathcal{G}(t)^4] \quad (3)$$

$$(\tanh^6): -5A_1(t)\mathcal{G}(t) \cdot [24\alpha\mathcal{G}(t)^4 + 20\gamma\mathcal{G}(t)^2A_2(t) + 3rA_2(t)^2] \quad (4)$$

$$(\tanh^5): -2\mathcal{G}(t)[-840\alpha A_2(t)\mathcal{G}(t)^4 + 10\mathcal{G}(t)^2\gamma A_1(t)^2 + 12\mathcal{G}(t)^2\alpha A_2(t) + 24\mathcal{G}(t)^2\gamma A_0(t)A_2(t) - 96\gamma A_2(t)^2\mathcal{G}(t)^2 - 3rA_2(t)^3 + 6rA_1(t)^2A_2(t) + 6rA_0(t)A_2(t)^2 + 2qA_2(t)^2] \quad (5)$$

$$(\tanh^4): -A_1(t)\mathcal{G}(t)[3rA_1(t)^2 - 240\alpha\mathcal{G}(t)^4 + 6\mathcal{G}(t)^2\mu - 184\gamma\mathcal{G}(t)^2A_2(t) + 12\mathcal{G}(t)^2\gamma A_0(t) + 18rA_0(t)A_2(t) + 6qA_2(t) - 15rA_2(t)^2] \quad (6)$$

$$(\tanh^3): -2A_2(t)\frac{d}{dt}\mathcal{H}(t) - 1232\alpha A_2(t)\mathcal{G}(t)^5 + 12rA_1(t)^2A_2(t)\mathcal{G}(t) - 2A_2(t)\frac{d}{dt}\mathcal{G}(t)x + 12rA_0(t)A_2(t)^2\mathcal{G}(t) - 6rA_0(t)A_1(t)^2\mathcal{G}(t) - 6rA_0(t)^2A_2(t)\mathcal{G}(t) - 2qA_1(t)^2\mathcal{G}(t) - 4qA_0(t)A_2(t)\mathcal{G}(t) + 80\gamma A_0(t)A_2(t)\mathcal{G}(t)^3 + 40\mu A_2(t)\mathcal{G}(t)^3 - 112\gamma A_2(t)^2\mathcal{G}(t)^3 + 32\gamma A_1(t)^2\mathcal{G}(t)^3 + 4qA_2(t)^2\mathcal{G}(t) \quad (7)$$

$$(\tanh^2): -A_1(t)\frac{d}{dt}\mathcal{H}(t) + 3rA_1(t)^3\mathcal{G}(t) + \frac{d}{dt}A_2(t) + 8\mu A_1(t)\mathcal{G}(t)^3 + 16\gamma A_0(t)A_1(t)\mathcal{G}(t)^3 - A_1(t)\frac{d}{dt}\mathcal{G}(t)x - 136\alpha A_1(t)\mathcal{G}(t)^5 + 18rA_0(t)A_1(t)A_2(t)\mathcal{G}(t) - 3rA_0(t)^2A_1(t)\mathcal{G}(t) + 6qA_1(t)A_2(t)\mathcal{G}(t) - 2qA_0(t)A_1(t)\mathcal{G}(t) - 92\gamma A_1(t)\mathcal{G}(t)^3A_2(t) \quad (8)$$

$$(\tanh^1): 2A_2(t)\frac{d}{dt}\mathcal{H}(t) + 272\alpha A_2(t)\mathcal{G}(t)^5 - 32\gamma A_0(t)A_2(t)\mathcal{G}(t)^3 + 2A_2(t)\frac{d}{dt}\mathcal{G}(t)x + 6rA_0(t)A_1(t)^2\mathcal{G}(t) + 6rA_0(t)^2A_2(t)\mathcal{G}(t) + \frac{d}{dt}A_1(t) - 16\mu A_2(t)\mathcal{G}(t)^3 + 2qA_1(t)^2\mathcal{G}(t) + 16\gamma A_2(t)^2\mathcal{G}(t)^3 + 4qA_0(t)A_2(t)\mathcal{G}(t) - 12\gamma A_1(t)^2\mathcal{G}(t)^3 \quad (9)$$

$$(\tanh^0): A_1(t)\frac{d}{dt}\mathcal{G}(t)x + A_1(t)\frac{d}{dt}\mathcal{H}(t) - 2\mu A_1(t)\mathcal{G}(t)^3 + 16\alpha A_1(t)\mathcal{G}(t)^5 + 3rA_0(t)^2A_1(t)\mathcal{G}(t) + 2qA_0(t)A_1(t)\mathcal{G}(t) + 8\gamma A_1(t)\mathcal{G}(t)^3A_2(t) - 4\gamma A_0(t)A_1(t)\mathcal{G}(t)^3 + \frac{d}{dt}A_0(t) \quad (10)$$

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We find the conditions for $\mathcal{A}_N(t)$, $\mathcal{G}(t)$, and $\mathcal{H}(t)$ which simultaneously cause the above terms to become zero:

$$(\tanh^1 \cdot x): \mathcal{A}_2(t) \frac{d}{dt} \mathcal{G}(t) x = 0 \quad (11)$$

$$\rightarrow \mathcal{G}(t) = \mathcal{G} = \text{non-zero constant},$$

$$(\tanh^4): \mathcal{A}_0(t) = -\frac{38\gamma^2 \mathcal{G}^2 + r\mu}{2r\gamma}, \quad (12)$$

$$(\tanh^7 \text{ and } \tanh^6): \mathcal{A}_2(t) = -6 \frac{\gamma \mathcal{G}^2}{r}, \quad (13)$$

$$(\tanh^5): \mathcal{A}_1(t) = \frac{12\gamma \operatorname{csgn}(\mathcal{G}) \mathcal{G}^2}{r}, \quad (14)$$

where $\operatorname{csgn}(x)$ is defined by

$$\operatorname{csgn}(x) = \begin{cases} 1 & \text{if } \operatorname{Re}(x) > 0 \\ -1 & \text{if } \operatorname{Re}(x) < 0 \end{cases} \quad (15)$$

$$(\tanh^0): \mathcal{H}(t) = \frac{(3\mathcal{G}\mu^2 r^2 + 300\mathcal{G}^3 \mu \gamma^2 r + 6924\mathcal{G}^5 \gamma^4)t}{5\gamma^2 r} + \frac{4}{5} \mathcal{C}, \quad (16)$$

where $\mathcal{C}$ is an arbitrary constant. In addition to the above conditions, the following auxiliary condition is required for $v(x, t)$ to be the solution of (1):

$$q = \frac{3}{2} \frac{r\mu + 50\gamma^2 \mathcal{G}^2}{\gamma}. \quad (17)$$

Thus, computerized symbolic computation helps us to obtain a new family of exact solitary-wave solutions of (1), i.e.

$$v(x, t) = \frac{12\gamma \mathcal{G}^2}{r} \cdot \tanh \left[\mathcal{G}x + \frac{(3\mathcal{G}\mu^2 r^2 + 300\mathcal{G}^3 \mu \gamma^2 r + 6924\mathcal{G}^5 \gamma^4)t}{5\gamma^2 r} + \frac{4}{5} \mathcal{C} \right] - 6 \frac{\gamma \operatorname{csgn}(\mathcal{G}) \mathcal{G}^2}{r} \cdot \tanh^2 \left[\mathcal{G}x + \frac{(3\mathcal{G}\mu^2 r^2 + 300\mathcal{G}^3 \mu \gamma^2 r + 6924\mathcal{G}^5 \gamma^4)t}{5\gamma^2 r} + \frac{4}{5} \mathcal{C} \right] - \frac{38\gamma^2 \mathcal{G}^2 + r\mu}{2r\gamma}. \quad (18)$$

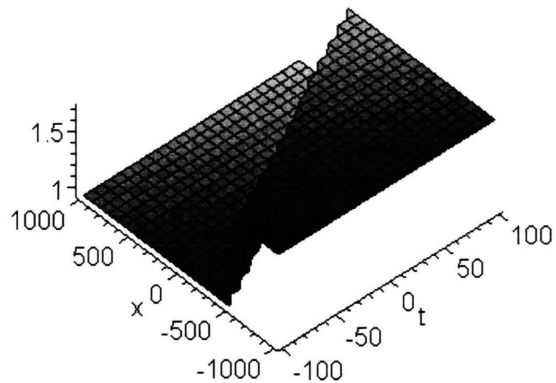


Fig. 1. A kink-typed solitary wave with the parameters $\mu=1$, $\gamma=1$, $q=1$, $\mathcal{C}=1$, $r=-1$, and $\mathcal{G}=\sqrt{-18r+12/30}$.

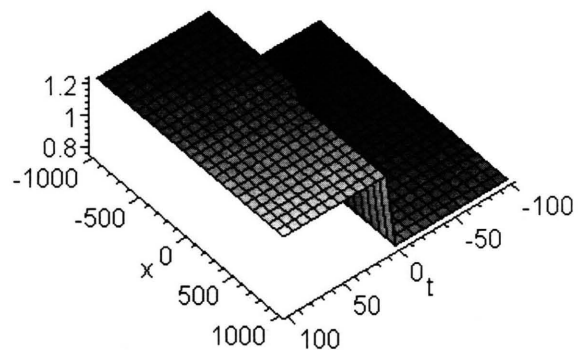


Fig. 2. A kink-typed solitary wave with the parameters $\mu=1$, $\gamma=1$, $q=1$, $\mathcal{C}=1$, $r=-1000$, and $\mathcal{G}=\sqrt{-18r+12/30}$.

Lastly we present figures with the some selected parameters. For simplicity we set $\mu=1$, $\gamma=1$, the auxiliary condition $q=\frac{3}{2}r+75\mathcal{G}^2=1$, and $\mathcal{C}=1$. In Figs. 1 and 2 we plot the kink-typed solitary waves with parameters: $r=-1, -1000$, $\mathcal{G}=\sqrt{-18r+12/30}$, respectively.

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